Length Varied Coplanar Angle Solar Panel Calculations

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In my previous exploration, "*Are Recessed Coplanar Solar Panels Better than Monoplanar Panels?*", I explored a simplified model for calculating the solar-panel efficiency of a monoplanar design versus a coplanar design. In the results the coplanar design with an angle of 120 degrees created a 3.38% gain over a monoplanar design, however it required about 1.547 as much panel surface-area which creates a cost-inefficient design. One consideration proposed in the closing sections of that paper were to vary the angle (denoted θ) over the length of the panel, which will be what this piece focuses on exploring.

Review of Formula

To calculate the efficiency of a monoplanar panel the formula is,

$$E = \epsilon * \psi 3600 * (\alpha\beta) * t$$

where,

1. E is the power produced over t hours of high-irradiance sunlight.
2. ϵ is the solar panel conversion efficiency, which is how efficiently it turns solar energy into electrical energy. The constant of 0.18 was used in the previous paper, and will be used here again.
3. ψ is the power production in watt-hours, with common panels producing about 5kWh, or 5000 watt-hours. This is multiplied by 3600 as this converts to the net amount of watts produced per-hour.
4. The panel has a width α and length β, which can be multiplied together to get the surface-area (γ)

The example monoplanar panel described was 2m by 4m, for a total surface-area of 8 square-meters. Given a 5kWh rating at 0.18 efficiency, this produced 25,920,000 watts over an hour of full sunlight. The prior considerations were over 8 hours of good sunlight, so the example monoplanar panel produced 207,360,000 watts, or 207.36 megawatts of power per-day.

To calculate the dimensions of a coplanar panel the formula is,

$$t' = \frac{\theta}{\pi} * (8 * 3600) = \frac{\theta * 28800}{\pi}$$
$$\theta_h = \frac{\pi - \theta}{2}$$
$$\alpha' = \frac{\alpha}{cos(\theta_h)}$$
$$h = \alpha' * sin(\theta_h) = \alpha * tan(\theta_h)$$
$$\gamma = \alpha' \beta$$

Where,

1. θ is the angle between the two panels.
2. θ_h is the angle of elevation of a panel.
3. α' is the adjusted length of the panel that sits over a width of α, which in a coplanar panel is 1/2 the total panel width. Keep this in mind, it is 1/2 what α is in a monoplanar design.
4. h is the height of the adjusted panel.
5. t' is the adjusted efficienct sunlight window.

and to calculate the efficiency the formula is very similar to the monoplanar formula of,

$$E = \epsilon * \psi 3600 * 2(\alpha' \beta) * t'$$

In the previous paper an angle of 120 degrees was used. This created a coplanar panel structure which was efficient for 5 hours and 20 minutes, and produced around 214 megawatts of energy.

Variable Length Coplanar Arrays

The question proposed at the end of the array was to a variation of the angle between the coplanar arrays along the length and how much of a gain in efficiency and power-production such a change could produce. The reasoning behind this is biomimetically derived from leaves, of which some species have varying angles along their length. This variation is between two extremes, with one end being flat ($\theta = 180^\circ = \pi$) and the other at an established maximum angle. To avoid confusion, the flat-angle will be denoted θ_0 and the non-flat angle will be denoted θ_n. The angle will linearly interpolate along the length of the solar panel between θ_0 and θ_n. Linear interpolation is done with the following formula,

$$lerp(A, B, \alpha) = A(1 - \alpha) + B\alpha$$

Which is used in this circumstance as,

$$\theta_i = lerp(\tau_i) = \theta_n(1 - \tau_i) + \theta_0\tau_i$$

where τ_i is the percentage from one end to the other a point, i, is on the solar panel, which is calculated by the formula,

$$\tau_i = \frac{p_i}{\beta}$$

given, p_i is the length from the edge of the panel where θ_1 is the coplanar angle. This expands out to,

$$\theta_i = \theta_n(1 - \frac{p_i}{\beta}) + \theta_0\frac{p_i}{\beta}$$

Given the surface-area of a segment is influenced by α' which changes for each coplanar angle of a segment, we will need to split the calculation the whole-panel's energy production into the energy production of each segment with a distinct coplanar angle. If this angle is continuously changed, creating a smoothly curved panel, the integration would be outside the scope of this paper, and so a simplified model will be used instead. The 4m long panel will instead be split into 4 segments, over which 4 different energy calculations will be done. The 1st segment will be of value θ_n and the last of value θ_0.

Given $\theta_n = 120^\circ = \frac{2\pi}{3}$ and $\theta_0 = 180^\circ = \pi$, the following table shows the θ_i value for each of the 4 segments.

Number	Angle (Degrees)

Number	Angle (Degrees)
1	120
2	140
3	160
4	180

Each of these segments will take up 1m of the length of the solar panel. Given these values of θ_i we can calculate the dimensions of each segments θ_h value, as given the following table

Number	Angle
1	pi/6
2	pi/9
3	pi/18
4	0

[1] https://www.wolframalpha.com/input?i=(pi+-+a)%2F2+given+a+%3D+[120pi%2F180%2C+140pi%2F180%2C+160pi%2F180%2C+180pi%2F180]

Which allows us to calculate the adjusted widths of each panel (α_i'),

Number	Adjusted Width
1	1.547
2	1.064
3	1.015
4	1

[1] https://www.wolframalpha.com/input?i=1%2F(cos(pi%2F6))

[2] https://www.wolframalpha.com/input?i=1%2F(cos(pi%2F9))

[3] https://www.wolframalpha.com/input?i=1%2F(cos(pi%2F18))

[4] https://www.wolframalpha.com/input?i=1%2F(cos(0))

Given these, and knowing that $\beta = 1m$ for each segment, we know that each of the panel segments represent their respected adjusted-widths in square-meters per-panel.

The next value needed is to calculate the peak effective hours for each pair of segments given their t_i' value,

Number	Adjusted Peak Hours
1	16/3 = 5.333…333
2	56/9 = 6.222…2222
3	64/9 = 7.111…111
4	8

[1] https://www.wolframalpha.com/input?i=8(a%2F180)+given+a+%3D+[+120%2C+140%2C+160%2C+180+]

These values can then be combined to compute the power generated per-day for each segment,

$$
\begin{gathered}
given\ \epsilon * (\psi 3600) = 3240000 \frac{W}{m^2} = \nu \\
E_1 = 3240000 \frac{W}{m^2} * 2(1.547m^2)\frac{16}{3} \\
E_2 = 3240000 \frac{W}{m^2} * 2(1.064m^2)\frac{56}{9} \\
E_3 = 3240000 \frac{W}{m^2} * 2(1.015m^2)\frac{64}{9} \\
E_4 = 3240000 \frac{W}{m^2} * 2m^2 * 8
\end{gathered}
$$

Which provides the following table of values,

Numbers	Power (Watts)
1	53,464,320
2	42,900,480
3	46,771,200
4	51,840,000

[1] https://www.wolframalpha.com/input?i=3240000+*+2(1.547)+*+(16%2F3)

[2] https://www.wolframalpha.com/input?i=3240000+*+2(1.064)+*+(56%2F9)

[3] https://www.wolframalpha.com/input?i=3240000+*+2(1.015)+*+(64%2F9)

[4] https://www.wolframalpha.com/input?i=3240000+*+2(1)+*+8

That can then be summed together to get the net-generated value of 194976000 Watts per day, or 194 megawatts.

Conclusion

Compared to the 214 mW produced by the fixed 120 degree coplanar panel, and the 207 mW produced by a flat monoplanar panel, the 194 mW produced by the variable angle coplanar panel is lower. This panel also still requires more material than the flat-panel design, and therefore it appears to be that the adjusted per-hour efficiency is all-together too low to be worthwhile. However, this does illustrate that there is a non-linear relationship between the angle and power generation curve as the two extrema of the provided angles show the most cost-efficient behaviors, while the middle values do not.

Therefore it may be useful to construct formula demonstrating this relationship, followed by running calculations to identify optimal angles and attempting to model if variable angle segments with only optimal angles can create an ideal trade-off. Mathematically this would imply that the weighted-sum of the time-by-power coefficients would have to be better than a single time-by-power coefficient. This may fall caveat if one only creates a derivative net power-coefficient, and so computer simulation may be a better approach in such an undertaking.

Overall, this does suffice to demonstrate that certain angles are better than others, and that a simple linearly interpolated coplanar design is less optimal than a simple coplanar or monoplanar design.