Are Recessed Coplanar Solar Panels Better than Monoplanar Panels?

By Joseph Juma

I want to explore whether or not, from a strictly mathematical standpoint, it may be more beneficial to deploy recessed coplanar solar panels over monoplanar designs. First, what is coplanar versus monoplanar?

1. Monoplanar is just a single planar panel, which is the most common form that solar panels come in.
2. Coplanar is to have a solar panel that is split into two smaller planes, each of which is oriented based on an angle between them. If the angle was about 30 degrees, they may look like a V.

The factors that may make coplanar more efficient are based on two basic principles. First, solar panels operate by exposing a photoelectric material to light from the sun to induce current in a device. Then there is the simple geometric phenomenon where a diagonal plane can fit a larger surface area into the same flat-length than a flat panel. Combining these two aspects a diagonal panel should increase the surface area exposure of light and increase panel efficiency.

Clearly this has played some part in Solar Panel design already as well, as almost no solar arrays sit flat, and many are oriented in some diagonal direction. Some more sophisticated arrays even have actuators which can change their angle, however this consumes power and so it can reduce the net benefit of the array overall.

Mathematics

We know that solar panels have a linear relationship between surface-area, which will be denoted by a, and power production, denoted by b, mediated by the amount of light based energy received. Thankfully, there is a common metric known as `solar irradiance` which is the amount of energy sunlight produces per unit of surface area at a given light frequency. Solar irradiance is measured in watts per square-meter, and we know that 1 watt is equal to 1 joule of energy per second, so solar irradiance can easily be converted into joules per square-meter per second.

When measuring solar irradiance, there are a few different types of radiation types measured, however the most useful one for establishing the best possible operation of a solar panel is `direct normal irradiance` (DNI) which is the measure of beams directly coming from the sun and only losing energy due to mostly atmospheric conditions like atmospheric scattering and other factors like fog. Given the following map,

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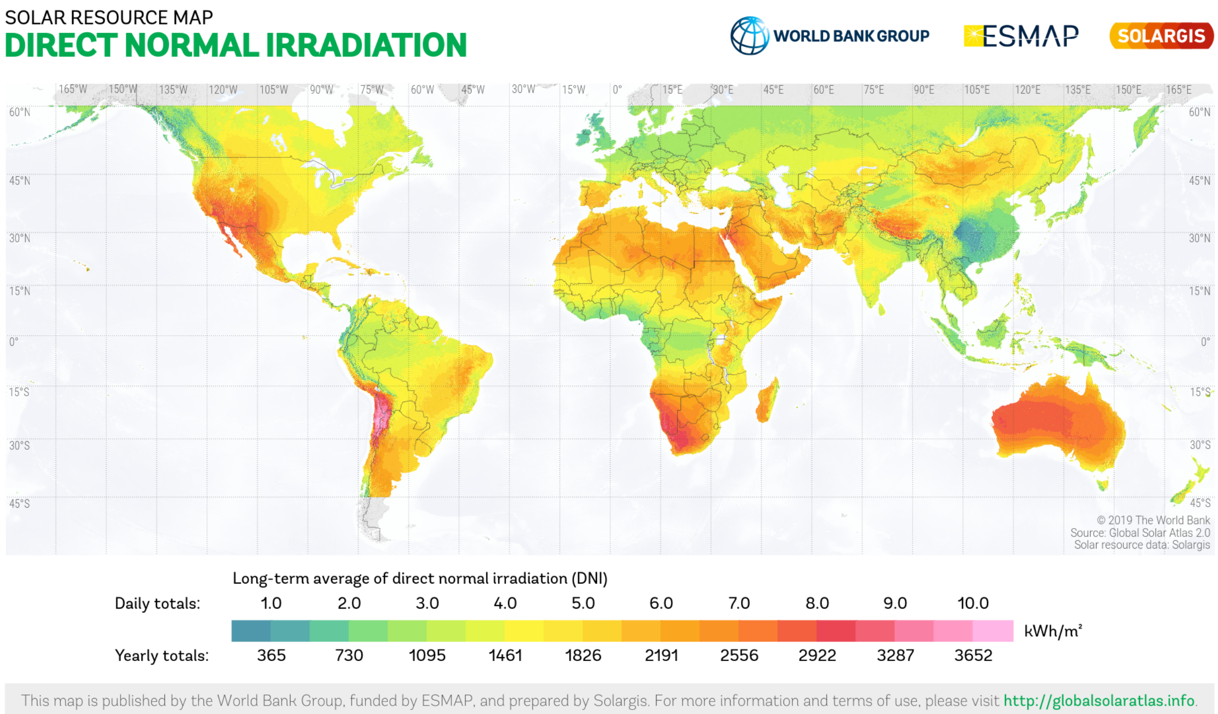
Figure 1.1



Courtesy of Wikimedia, https://upload.wikimedia.org/wikipedia/commons/6/65/World_DNI_Solar-resource-map_GlobalSolarAtlas_World-Bank-Esmap-Solargis.png

We can note that most land on Earth is in the range of between 3 and 7 kWh per square-meter, which averages out to 5 kWh per square-meter. A watt-hour is a flow of 1 watt of power, or 1 joule per-second, happening over an entire hour, for a total of 3600 joules of net energy. 5 kWh is equal to 5000 joules of energy every second over an hour, or 18,000,000 joules of energy over an hour. For clarity, the equation is as follows,

Equation 1.1

$$\begin{gathered} given\ E\ is\ energy \\ given\ W\ is\ power \\ E = W3600 \end{gathered}$$

In this preliminary estimation, we will not account for things like seasonal change, elevation changes, local geography or weather and irradiance hours per-day. All of these values should be later accounted for in a more complete model if this simplified one shows promise.

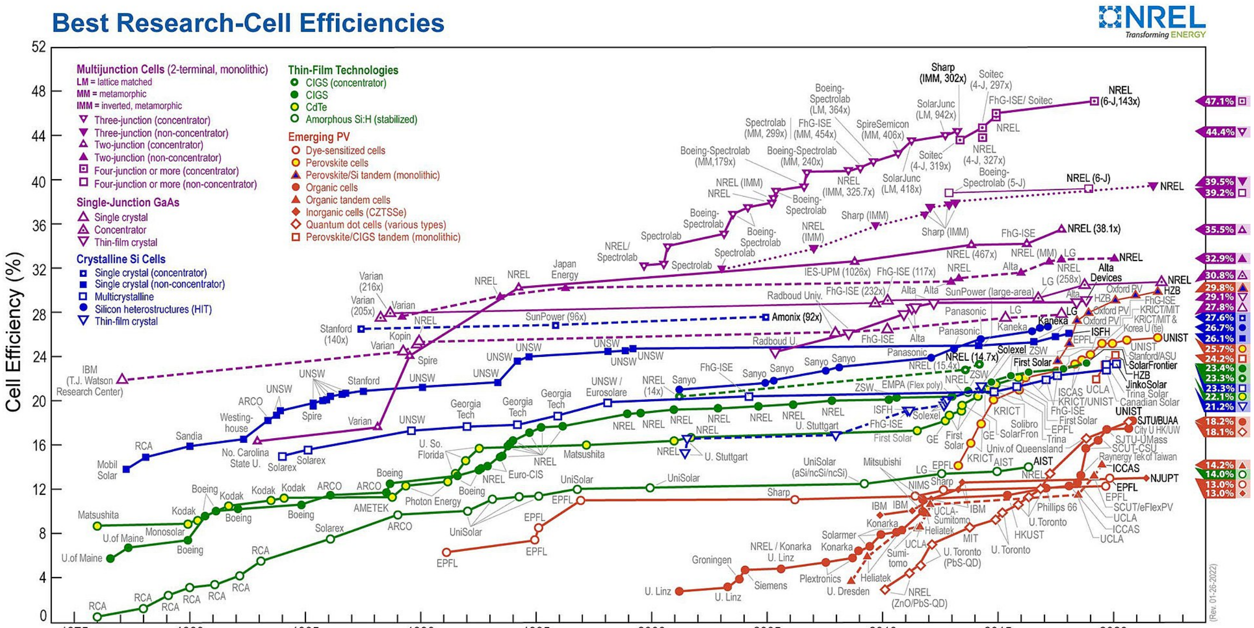
If we calculate that there are on average 8 total useful hours of day-light per day, then we can calculate that the average solar irradiance under DNI for Earth's land-mass as,

Equation 1.2

$$E = (5000\frac{j}{sm^2} * 3600s) * 8 = 144\frac{j}{m^2} * 10^6$$

For each unit of solar irradiance, the solar panels will have some linear conversion factor, denoted by α which expresses how much energy a panel outputs given some amount of input, as solar panels are not 100% efficient in the transformation of light-based energy into electrical energy. Given the following diagram,

Figure 1.2



Courtesy of Wikimedia, https://en.wikipedia.org/wiki/Solar_cell_efficiency#/media/File:Best-research-cell-efficiencies-rev220126_pages-to-jpg-0001.jpg

We can see that at the time of writing, August 2022, research solar cell efficiencies range between 13% and 47%, but we will use a lower-end cell as these are common for the market. 20% appears to be a common consumer-grade solar panel efficiency, which is at the far lower end of the research cell spectrum, allowing for improvements in efficiencies to increase throughput of these projections further down the line.

Given this value we can establish that for a given solar panel that is 1 square-meter in surface area, over an 8-hour day we will see an average output energy given by the following formula,

Formula 1.3

$$E = 0.18 * (5kWh) * 8h = 7.2kWh = 7200Wh = 7200Wh * (3600s) = 25,920,000j$$

So a single square-meter can produce 25.92 megajoules of energy per day. The question is then, what is the increase in surface-area given a coplanar design versus a monoplanar design?

Monoplanar Configuration

For the monoplanar configuration we will use a flat panel, lain flat, that is 2 meters by 4 meters for a total of 8 square-meters. With the above equation, we can compute that this panel will output a total of 207,360,000 joules per-day. This is straight-forward since we have already been doing math based on monoplanar configurations.

Coplanar Configuration

The coplanar configuration will also take up 2 meters by 4 meters, but being angled will have a greater surface-area. This configuration can be thought of as two separate panels, each taking up 1 meter by 4 meters (4 square-meters). As these two planes will only be tilted on one shared axis, described by the angle between their two planes, we can use simple trigonometry and geometry to calculate the actual panel surface area. The axis of raising will be along the 1m edge, rather than the 4m edge.

Given the angle between two panels (θ), we can know that we can calculate the height by dividing the angle in-half so as to isolate one side, and this new angle will be the complimentary angle to the angle of raising of the panel. We can subtract this angle ($\frac{\theta}{2}$) from 90 degrees ($\frac{\pi}{2}$) to calculate the height of a raised panel (y) using the following formula,

Formula 2.1

$$y = r * sin(\frac{\pi}{2} - \frac{\theta}{2}) = r * sin(\frac{\pi - \theta}{2})$$

From here we need the value of r, which can be calculated via the formula,

Formula 2.2

$$x = r * cos(\frac{\pi - \theta}{2}) \rightarrow r = \frac{x}{cos(\frac{\pi-\theta}{2})}$$

Therefore we can combine these formulas to get,

Formula 2.3

$$y = \frac{x}{cos(\frac{\pi-\theta}{2})} * sin(\frac{\pi - \theta}{2}) = x * \frac{sin(\frac{\pi-\theta}{2})}{cos(\frac{\pi-\theta}{2})} = x * tan(\frac{\pi - \theta}{2})$$

We know that x is equal to one square-meter, and we can set θ to 120* (or 30* elevation per-panel), so we can calculate the actual length of the panels and their elevation as,

Formula 2.4

$$r = \frac{1}{cos(\frac{\pi-\theta}{2})} = cos(\frac{\pi - \frac{2\theta}{3}}{2})^{-1} = cos(\frac{\pi}{6})^{-1} = (0.8660254...)^{-1} = 1.1547005... = 1.547$$
$$y = tan(\frac{\pi}{6}) = 0.577350... = 0.5774$$

[1] https://www.wolframalpha.com/input?i=1%2Fcos((pi+-+(2*pi%2F3))%2F2)

[2] https://www.wolframalpha.com/input?i=tan(+pi%2F6+)

So, given an elevation of 0.5774 meters, we're able to increase the width of the panel from 1m to 1.547m, increasing the surface-area from 4 square-meters per panel, to 6.188 square-meters, which brings the total coplanar panel to a total of 12.376 square-meters from 8 meters.

We then need to consider that there will be sub-optimal and optimal subsets of time in the 8-hour span we consider for solar power collection for each of the two panels based on their angling. We will consider that a full 8-hour cycle is 100% light-efficient for the full-duration. We can also assume that when the sun is within the central θ region of the sky it is also 100% efficient as a line can be directly drawn to all points on both panel surfaces while the sun is within this arc of the half-circle that constitutes the 8-hour of irradiance.

To calculate the percentage of time the optimal arc takes up, we can just take the percentage of 180* that the angle takes up and multiply them by 8. Given $\theta = 120degs$ this is $\frac{2}{3}$ or 0.666…667 = 0.6667, which equates to 5 hours and 20 minutes, or 5.333 hours. This will be called the `critical period`. Outside of the critical period, only one-half of the solar panel will receive some amount of light, which will decreased based on the projected arc of the sun, occluded by the arc projected by the panel not facing the sun. However, calculating this occlusion is outside the scope of the current exercise, and so anything outside the critical period will be counted as 0 power generated. This reduction in complexity means that the lower-bound calculated with this projection will be less than the real output.

With these parameters, we can adjust the calculation from above,

Figure 2.5

$$E = 12.376m^2 * 0.18 * (5\frac{kWh}{m^2} * 5.3333h * 3600s) = 2.22768m^2 * (96000000\frac{J}{sm^2}) = 213857280\frac{J}{s} = 213.85728 *$$

This comes out to about 214 megawatts, in comparison to the 207 megawatts produced by the monoplanar design. If we truncate these values to the megawatt, this marks the lower bound as a 3.38% gain in energy generated. This shows this approach *does* improve the efficiency of a solar array.

Caveats

1. Although the above does show an increase in solar output, there is a cost-to-benefit ratio when considering real-world deployments. This is an increase in surface-area, and therefore solar-panel material of 1.547x, increasing costs by almost 1.5, for only a 3.38% gain in power.
2. This calculation extrapolates out a large number of variables. Some of these variables would increase the efficiency of the system, and others would severely reduce it.

Given these two caveats, one should consider further investigations of similar approaches which may improve this solar panel design.

Further Proposals

- What if we alter the angle (θ) over the length of the coplanar panel, so some segments have a steeper angle, and others have a nearly flat angle?
- What if we use layering of solar panels? Three-prong leaves allow for gaps between them, which could be causing a pin-hole effect (optics) that allows light to reach leaves beneath them at better projection angles. This could mean we use a pinhole effect (by having gaps in a panel) which then allows panels underneath to also absorb light.

Conclusion

With this information we can conclude that although there are marginal gains, further proposals and calculations will be done to increase the cost-effectiveness of such an approach over the current mono-planar designs. Two such considerations are variable-angle coplanar designs, and pinhole layering approaches to better use planetary surface-area for maximized power efficiency. Overall the possible approaches are numerous and showing early potential for promising innovations.